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Energy consideration from non-equilibrium to equilibrium state in the process of charging a capacitor

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Abstract

A calculation of the energy loss due to a transition from non-equilibrium to equilibrium state in the process of charging a capacitor is given. In this study, it is shown that half of the total energy produced by the battery should be consumed in one form or another to reach the equilibrium state, regardless of the values of the resistance and capacitance.

Keywords: Electrostatic energy; Charging a capacitor; Equilibrium from non-equilibrium; Energy loss

1. Introduction

It is well known that energy is a very important fundamental concept in physics. The total energy of a system can be subdivided and classified in different forms: kinetic energy, potential energy, electromagnetic energy, nuclear energy, etc. According to the law of conservation of energy, energy can only be transformed from one form to another. The only way the energy of a given system can change is by carrying energy, in several ways, into or out of that system. Every day we use electrical or mechanical devices that change energy from one form to another. In a battery, for example, chemical energy is transformed to electric energy; and in a lamp electrical energy is changed to light energy and heat, and so on. Therefore, energy is a concept which allows us to describe and understand many processes around us. In addition, it is a measure of the ability of a given system to produce changes in its own state as well as changes in the states of its surroundings [1].

To reach an equilibrium state, some amount of the initial energy for a given non-equilibrium system must be lost (i.e., transformed to its surroundings) in one form or another. However, a deeper look into the energy lost in process of transition from a non-equilibrium to an equilibrium state is often overlooked, especially by authors of introductory physics textbooks. such authors usually consider only the ideal case in problems dealing with the transformation of energy.

The aim of this paper is to show that when electrical energy from a battery is used to charge a capacitor, an amount of energy equal to that stored in the capacitor must be consumed in one form or another by the rest of the system regardless of the values of the parameters of the system. In the following section, we derive and present the energy lost in the process of charging a capacitor. We close the paper by summarizing the results and discussing their significance.

2. Calculations and discussions

In this section, the derivation of the energy lost due to a transition from a non-equilibrium to an equilibrium state in a process of charging a capacitor is given.

Assume that an initially uncharged ideal capacitor of value C is connected to an ideal battery of a constant electromotive force V_0 by some cylindrical metallic wires and switch S, as shown schematically in Fig. 1. At time $t=0$ the switch S is closed. After a very long time the system is assumed to reach the equilibrium state, in which a maximum electrostatic energy is stored in the capacitor.

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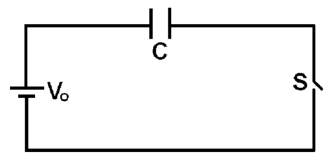

Fig. 1. A typical electrical circuit which consists of an ideal battery of a constant electromotive force V_0, an ideal capacitor C, a switch S, and some cylindrical metallic wires.

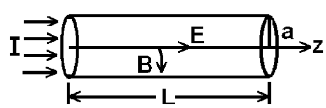

Fig. 2. A current I flowing through a long cylindrical metallic wire of radius a and length L.

The connection wires shown in Fig. 1 are considered to be a one effective wire, which is a long cylindrical metallic wire of length L and radius a, as shown in Fig. 2.

At any time t, the electric field $\vec{E}$ inside the wire and the magnetic field $\vec{B}$ at its surface are given by [2]

$$\vec{E} = \frac{V_0 - V_c}{L}\hat{z} \tag{1}$$

and

$$\vec{B} = \frac{\mu_0 I}{2\pi a}\hat{\phi}, \tag{2}$$

where V_c is the potential difference across the capacitor and I is the charging current.

The integration of the Poynting vector

$$\vec{S} = \vec{E} \times \frac{\vec{B}}{\mu_0} = -\frac{(V_0 - V_c)I}{2\pi a L}\hat{\rho} \tag{3}$$

over the closed surface of the wire, using the cylindrical coordinates (ρ, ϕ, z), gives the power of the electromagnetic energy (P_f) flow into the wire through its closed surface as

$$P_f = -\oint \vec{S}\cdot d\vec{A},$$

$$\begin{aligned} P_f &= -\int_{\text{round surface}} \vec{S}\cdot d\vec{A} \\ &= \int_0^{2\pi}\int_0^L \frac{(V_0 - V_c)I}{2\pi a L} a\,dz\,d\phi \\ &= V_0 I - V_c I \\ &= P_b - P_c \\ &= \frac{d}{dt}(V_0 q(t)) - \frac{d}{dt}\left(\frac{q^2(t)}{2C}\right), \end{aligned} \tag{4}$$

where $q(t)$ is the charge on the capacitor at time t, P_b is the electric power provided by the battery, and P_c is the electric power flows in the capacitor, and dA is an element of area perpendicular to the surface of the wire in cylindrical coordinates. Eq. (4) shows that the difference between the rate of the energy produced by the battery and the rate of the energy stored in the capacitor equals the rate of the electromagnetic energy passing into the wire through its sides, no matter what values of the resistance and capacitance are.

To obtain the total *EM* energy transferred into the wire (U_f), we integrate Eq. (4) over the time interval $[0, \infty)$ as follows:

$$\begin{aligned} U_f &= \int_0^\infty P_f\,dt \\ &= \int_0^\infty P_b\,dt - \int_0^\infty P_c\,dt \\ &= q_0 V_0 - \frac{q_0^2}{2C} \\ &= CV_0^2 - \frac{1}{2}CV_0^2 \\ &= \frac{1}{2}CV_0^2. \end{aligned} \tag{5}$$

To achieve the equilibrium state, Eq. (5) shows that half the total energy provided by the battery is stored into the capacitor and half of it is passed into the connecting wire through its closed surface. The electromagnetic power flow into the wire may be manifested as an increase in mechanical, or potential, or chemical energy of the conducting wire [3–5]. Similar phenomenon occurs in the two-capacitor problem. For the case of the two-capacitor problem where the circuit has zero resistance, it was shown that the electromagnetic power radiates in the form of electromagnetic radiation [6–10].

Analogous situations persist in a widely well-known example in introductory physics courses [11,12], regarding the total work required to charge a spherical conducting shell of radius a to a total charge Q. A typical problem in most standard textbooks gives an answer of $Q^2/(8\pi\varepsilon_0 a)$ (for example, see problem 59 of Chapter 25 in Ref. [11]). This amount of energy should be the electrostatic energy stored in the electric field of the charged spherical shell, not the total work required to charge the spherical shell. A closer look at this problem shows that this problem is equivalent to charging an isolated spherical capacitor. According to the result presented above, the total work that must be done to charge the spherical shell is just twice the stored energy, namely $Q^2/(4\pi\varepsilon_0 a)$. Hence problems involving such examples in standard introductory textbooks, in particular those dealing with electromagnetism, may mislead students because, in reality, the process of charging an object should be done through a conducting medium which connects the object with the power source as seen in the process of charging a capacitor presented above. In order to avoid any confusion of physical concepts, it may be better to ask students to calculate the stored electrostatic energy in the electric field of the charged object rather than the work that must be done to charge that object.

We must emphasize that in the above discussed systems (and other similar examples) a constraint has been imposed on the process of transition from a non-equilibrium to an equilibrium state. The process of charging a capacitor, for example, it is usually assumed that the conducting wire between the capacitor and battery has a resistance R and all energy lost in this process is dissipated as a heat inside that wire. The present authors believe that there is no need to such a constraint because consumed energy, in one form or another, is actually the price that must be payed in order to achieve the final equilibrium state, regardless of the values for the parameters of the system.

3. Conclusions

In this paper we derive the energy lost in the process of charging a capacitor. The calculations show that half of the total energy produced by the battery will be stored in the capacitor, and half of it will be lost, regardless of the values of the resistance and capacitance. For the case of zero resistance, it can be shown that lost energy is radiated into space as electromagnetic waves.

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